

# Concept of Compact Highly-Sensitive Superconducting Antenna for Gravitational Wave Radiation

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## Abstract

A new concept for detectors of gravitational wave radiation is discussed. Estimates of its sensitivity suggest that these devices will be able to detect gravitational waves with amplitudes as low as  $h_0 \sim 10^{-26}$ . Such sensitivity could be obtained at spatial scales as small as 10 meters. Devices based on this concept require operational temperatures below the critical temperatures of their superconducting components.

## 1. Basic principle

Consider a superconducting bar of length  $L$  in the field of a gravitational wave (GW)  $h_{\mu\nu}(\mu, \nu=0,1,2,3)$ . For simplicity let  $h_{\mu\nu}$  be a plane wave with '+' polarization:  $h_{11}=-h_{22}=h$  [1] and let the bar be oriented along the  $x_1$  direction in the  $(x_1, x_2)$ -plane orthogonal to the direction of the wave propagation  $x_3$ . Take  $h$  to be given by:  $h(t)=h_0 \sin \omega t$ . To understand the influence of the GW on the superconducting bar, one should bear in mind that such a GW creates quadrupolar oscillations in a system of probe masses as shown in Fig. 1.

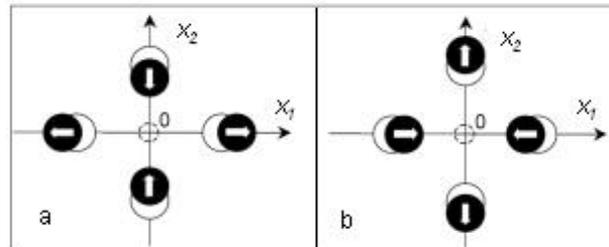


FIG.1. A plane gravitational wave propagating in the direction  $x_3$  induces a quadrupolar motion of test particles in the  $(x_1, x_2)$ -plane. The two panels correspond to consecutive half-periods of oscillations: a) shows the quadrupolar motion in (say) the first half-period; b) shows the quadrupolar motion in the second half-period.

The gravitational wave acts upon all the constituent masses in the bar. However, the resulting motion of the ions is very different from that of the electrons since the ions are bound whereas the superconducting electrons move freely without friction. Also the ionic system interacts with the external platforms/holders, which precludes their motion. Thus, the wave will tend to accelerate the electrons back and forth, towards and away from the ends of the bar along the  $x_1$ -axis (cf. Fig. 1, a and b) with the boundaries of the superconductor preventing the electrons (Cooper pairs) from leaving the bar. In practice, this means that due to the gravitational wave the phase  $\Theta$  of the wave function  $\Psi=\psi \exp(i\Theta)$  (which characterizes the state of superconducting electrons in the bar) will change. By symmetry, the phase will stay unchanged at the middle of the bar, and will be

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maximally changed (in the same direction) at the ends of the bar and this change will reverse its sign during the period of oscillation

Consider now a second adjacent, perpendicular, superconducting bar oriented along the  $x_2$ . As shown in Fig. 1, its oscillations will be shifted by a half-period, so that the phase shift of the wave function has a sign opposite that of the first bar. Thus, if one connects the ends of these bars, as in Fig. 2,

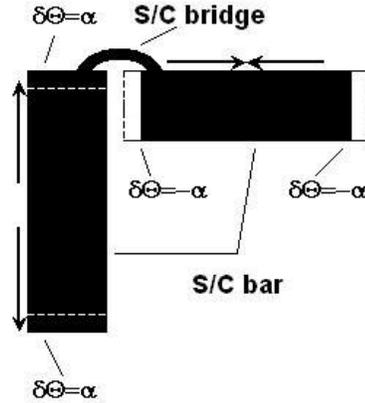


FIG.2. Underlying principle of the gravitational wave (GW) antenna. The quadrupolar action of the GW creates oscillating super-current through the bridge connecting orthogonal superconducting bars (the second half-period of Fig. 1 is shown).

by a short superconducting bridge, one would get an oscillating current (with frequency  $\omega$ ) through this bridge. This is the basic operational principle of the proposed gravitational wave antenna. It exploits the tensor nature of the field of a gravitational wave, which allows positive coupling between two orthogonal directions of motion. This will be crucial for the final design.

## 2. Simplified detector architecture

In working our way towards the concept of a working device let us first consider the more symmetric design of (Fig. 3).

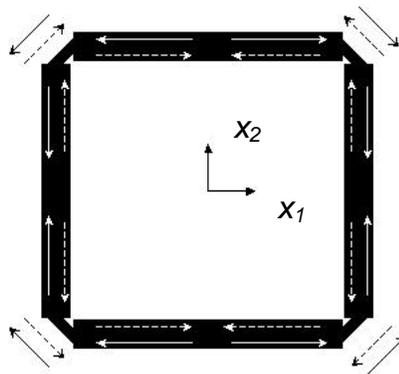


FIG.3. A “Symmetric” design of a GW antenna: two units (from Fig. 2) consisting of four superconducting bars. The white arrows indicate the directions of the driving force during the first half-period (solid lines) and the second half-period (dashed lines). Arrows indicate current flow. As most GW antenna, it clearly is directional – responding maximally to waves propagation in the  $x_3$  direction –orthogonal to the plane of the antenna.

This design was however introduced just as a preliminary starting point for further development. In its current form, it has a major disadvantage: under the influence of the gravitational wave the electrons move symmetrically- either from the centers to the ends of the bars or from the ends to the centers of the bars. The resulting concentration of electrons immediately creates Coulomb forces opposing this tidal action of the gravitational wave. Because the electromagnetic interaction is much stronger than the gravitational interaction, these Coulomb forces oppose and quench the tidal effects reducing the latter to a non-detectable level. One can call this phenomenon a “Coulomb blockade”.

### 3. A design free from Coulomb blockade

Fortunately, there is a way to prevent the Coulomb blockade. To this end let us consider a more sophisticated design (Fig. 4).

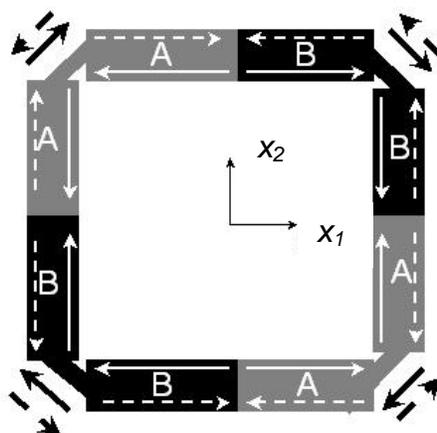


FIG.4. Bimetallic design: materials  $A$  and  $B$  have different parameters (see text), so that current always follows the way which is prescribed by  $B$ . White arrows indicate driving force directions during the first half-period (solid lines) and during the second half-period (dashed lines). Arrows indicate current flow; it is clockwise for the first half-period, and counter clockwise for the second half-period.

In this design, material  $B$  is different from material  $A$ : its' effective (inertial but not gravitational!) electron mass<sup>1</sup> is bigger ( $m_B > m_A$ ), and the charge carrier density is higher:  $n_S^B > n_S^A$ . So that the gravitational wave accelerates faster and more charge carriers in  $B$  than in  $A$ . The resulting motion will follow the direction preferred by  $B$  as carriers in  $A$  will be forced by the Coulomb interactions to follow the direction prescribed by the motion of the carriers in  $B$ ). No violation of electro-neutrality is expected for a current constant along the loop of the bars and no Coulomb blockade will arise in this close-circuit design.

### 4. Sensitivity

To assess the sensitivity of the detector shown in Fig. 4., let us estimate the resulting current. The bars are all of length  $L$  and cross-section  $S$ . Let us further assume that the current inside each bar are uniformly distributed over its cross-sectional area (the validity of this suggestion, as well as the ways to fulfill it, will be discussed in the next Section)  $S$ . For simplicity, one can estimate one

<sup>1</sup> The mechanical densities  $\rho_A$  and  $\rho_B$  should be close to each other, or otherwise the bar should be balanced to avoid the rotational effect.

$n_S^B = n_S^A = n_S$ . The driving force acting on the Cooper pairs due of the presence by the gravitational wave

$$h(t) = h_0 \sin \omega t \quad (1)$$

is equal to

$$F_{GW} = -m_{eff} x \omega^2 h_0 \sin \omega t, \quad (2)$$

where  $x$  denotes the coordinate along the bar  $-L/2 < x < L/2$ , and  $m_{eff}$  is the effective mass of the Cooper pair. This key expression for the force is analogous to the tidal force acting on a pair of free probe particles separated by a distance  $x$  from their center of mass:  $\ddot{x} = -x h_0 \omega^2 \sin \omega t$ ; in free space the force is then  $F_{GW} = -2m_0 x \omega^2 h_0 \sin \omega t$ , where  $m_0$  is the electron rest mass. In principle, one can start from this expression, and write down all other forces acting on the electron in the considered metal, including those resulting in superconductivity of electrons. This is a general recipe [2], however, the authors are not aware of any application of such recipe to superconductors or electrons in normal metals. A simpler way is to use free motion of the Cooper pairs in superconductors (so that the interaction with the lattice has already yielded renormalization of the electron mass) and use the expression (2) for further estimation of consequences imposed by electric neutrality. Application of Newtonian dynamics yields then, for the motion of a super-fluid electron liquid along the bar, a velocity amplitude

$$V = \frac{m_B - m_A}{m_B + m_A} \frac{L}{4} \omega h_0. \quad (3)$$

In the realistic case  $m_B = 2m_A = m$  (larger velocities and sensitivities follow if  $m_B \gg m_A$ ) the current through the bar (and actually, through all the bars) is:

$$I = eV n_S S = e n_S S L h \omega / 12. \quad (4)$$

For the antenna with parameters:  $L = 10^3 \text{ cm}$ ,  $n_S = 10^{23} \text{ cm}^{-3}$ ,  $S = 10^2 \text{ cm}^2$  the current resultant from Eq. (4) is about a Femto-Ampere for a wave with amplitude  $h_0 = 10^{-26}$  and frequency  $30 \text{ Hz}$ . These latter parameters have been chosen to tackle the detection of gravitational wave radiation from the Crab Nebula. The obtained estimates of sensitivity look quite inspiring, and deserve further discussion.

## 5. Discussion and Conclusions

In the above description, a bar with relatively big cross section  $S$  was considered. It assumes a correspondingly large lateral size, for example,  $10 \times 10 \text{ cm}^2$ . However, superconducting motion cannot take place in the depth of a superconducting object because of the Meissner effect. This situation may be compared with thermoelectricity. If one takes a metallic bar and heats up one end while keeping the other end cold, one will observe a motion of electric charges caused by the phonon drag. This motion will build up an electric voltage between the bar ends unless the metal is in the superconducting state, in which case the observable effects are much weaker: the motion of electric charges is allowed only within the London penetration depth  $\lambda_L$ , which is typically of the order of  $\lambda_L \sim 0.1 \mu\text{m}$  (a good discussion may be found in Ref. 3). In view of this remark one should

pursue an embodiment for the bar consisted of thin layers of superconducting metal separated by dielectric layers (Fig. 5).

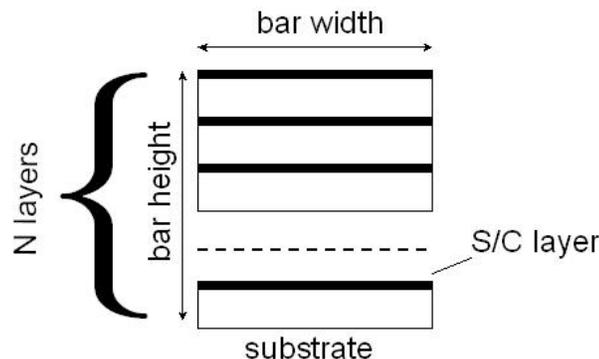


FIG.5. Cross- sectional view of the bar. A single layer of the superconducting material on top of the substrate is shown.

There is another problem which the layered design helps to overcome. That is the magnetic inductance of its electric loop. The energy absorbed from the gravitational wave creates not only the kinetic energy of current motion but also magnetic energy of the current. Using standard expressions for magnetic ( $E_{mag}$ ) and mechanical (kinetic,  $E_{kin}$ ) energies of the loop, one can obtain for their ratio

$$\frac{E_{mag}}{E_{kin}} = \frac{e^2}{4} \mu_0 \frac{n_s S}{m} \quad (5)$$

where  $\mu_0$  is the magnetic permittivity of vacuum, and  $S$  is the loop area. For our case the rough estimate provides for this ratio a factor  $\sim 10^{17}$ , which means that the energy of the gravitational field will be transferred to the magnetic inductance rather than to the motion of the charge. To avoid this problem the magnetic field of the loop should be strongly reduced, and that is possible in a layered structure: the current should move in opposite directions in consecutive layers. For that to be the case, the consecutive layers should have materials  $A$  and  $B$  swapped.

Measuring currents on the femto-ampere level is possible using off the shelf instruments [4]. Hewlett Packard, for example, offers a product (E5270B, Ref. 5) which can measure currents as low as hundred attoampere. Specifics of the measurement require a discussion; however that goes beyond this NOI, which is devoted to the description of the concept only.

Generally, superconductors are electrically quiet objects. However, certain thermodynamic noise reveals itself in superconductors as well. One should be aware that superconducting wave function phase noise is the major inherent noise source. It is possible to estimate the influence of this phase noise. This will be presented in future publications. The higher noise level will impose the longer observation time, which, however, is tolerable, as this antenna can be used to detect the periodic sources for long periods of time. Moreover, if the oscillation frequency is known, the noise can be effectively filtered out.

Related to the concept itself, in its final layout the antenna should be enclosed into a magnetic shield: a superconducting and/or  $\mu$ -metal enclosure to avoid magnetic disturbances, which also may cause noise.

Another aspect is the requirement of cooling of the system below the critical temperature of its superconducting components. That is a solvable task, even when the lateral sizes are tens of

meters. Again, it depends on the specifics of instrument application, and should be addressed for any specific case.

To summarize, a concept of solid-state gravitational wave antenna is proposed, which exploits the tensorial action of gravitational waves on the electrons in superconductors. The estimates demonstrate that very high sensitivity could be obtained in devices based on this concept. Relatively modest sizes of the antenna will allow their arrangement on space-borne low-noise platforms. Far from the Sun the devices will be passively cooled below critical temperature of their superconducting components so that there will be no necessity for onboard cryogenic coolers. Hopefully, in parallel to other large-scale efforts, such as the LIGO approach [6] and LISA mission [7] or NANO gravitational initiative [8], the suggested concept will become useful for one of the most challenging experiments - the detection of gravitational waves.

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